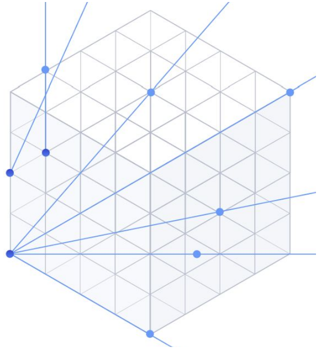
Technion
Israel Institute of Technology

Ramanujan Machine

Unifying Structure for Ramanujan's 17 Series for 1/π

A Human-AI Discovery

$$\frac{1}{\pi}=\frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$



Michael Shalyt, Elyasheev Leibtag, Shachar Weinbaum, Ido Kaminer.

From Euler to AI

$$\pi = \sum_{k=0}^{\infty} \left[\frac{1}{16^k} \left(\frac{4}{8k+1} - \frac{2}{8k+4} - \frac{1}{8k+5} - \frac{1}{8k+6} \right) \right]$$

Bailey-Borwein-Plouffe

Mathematics of Computation (1997)

$$\frac{\pi}{2} = \prod_{n=1}^{\infty} \frac{(2n)^2}{(2n-1)(2n+1)}$$

John Wallis

1656, England.

$$\pi = \cfrac{4}{1 + \cfrac{1^2}{3 + \cfrac{2^2}{5 + \cfrac{3^2}{7 + \ddots}}}}$$

Carl Friedrich Gauss

1813, Germany

$$\pi = \sqrt{12} \sum_{k=0}^{\infty} \frac{(-3)^{-k}}{2k+1}$$

Madhava of Sangamagrama

14th century, India.

$$\frac{4}{\pi - 2} = 3 + \cfrac{3}{5 + \cfrac{8}{7 + \cfrac{15}{9 + \cfrac{24}{11 + \ddots}}}}$$

Raayoni et al.

Nature (2021)

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801} \sum_{k=0}^{\infty} \frac{(4k)!(1103 + 26390k)}{(k!)^4 396^{4k}}$$

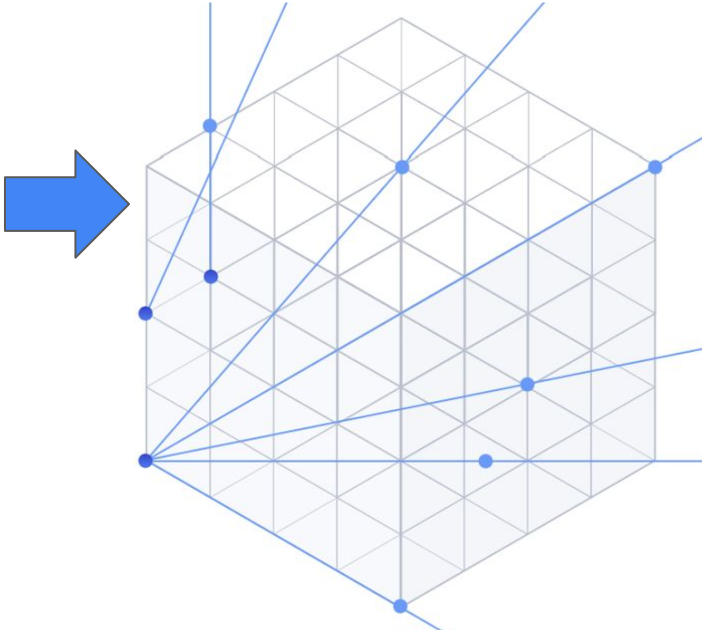
Srinivasa Ramanujan

1914, India.

Conservative Matrix Fields (CMF)

$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(c)_n} \frac{z^n}{n!} $$

$$ (a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} $$

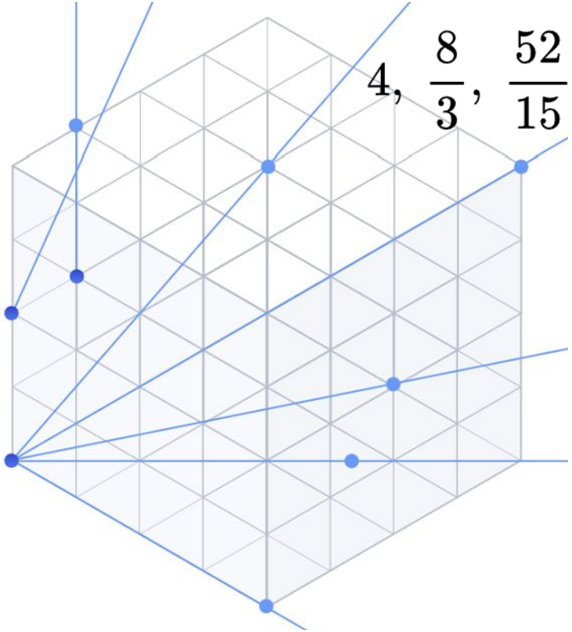


Elimelech et al. "*Algorithm-assisted discovery of an intrinsic order among mathematical constants*" (PNAS 2024)
Weinbaum et al. "*On Conservative Matrix Fields: Continuous Asymptotics and Arithmetic*" (arXiv:2507.08138)

Conservative Matrix Fields (CMF)

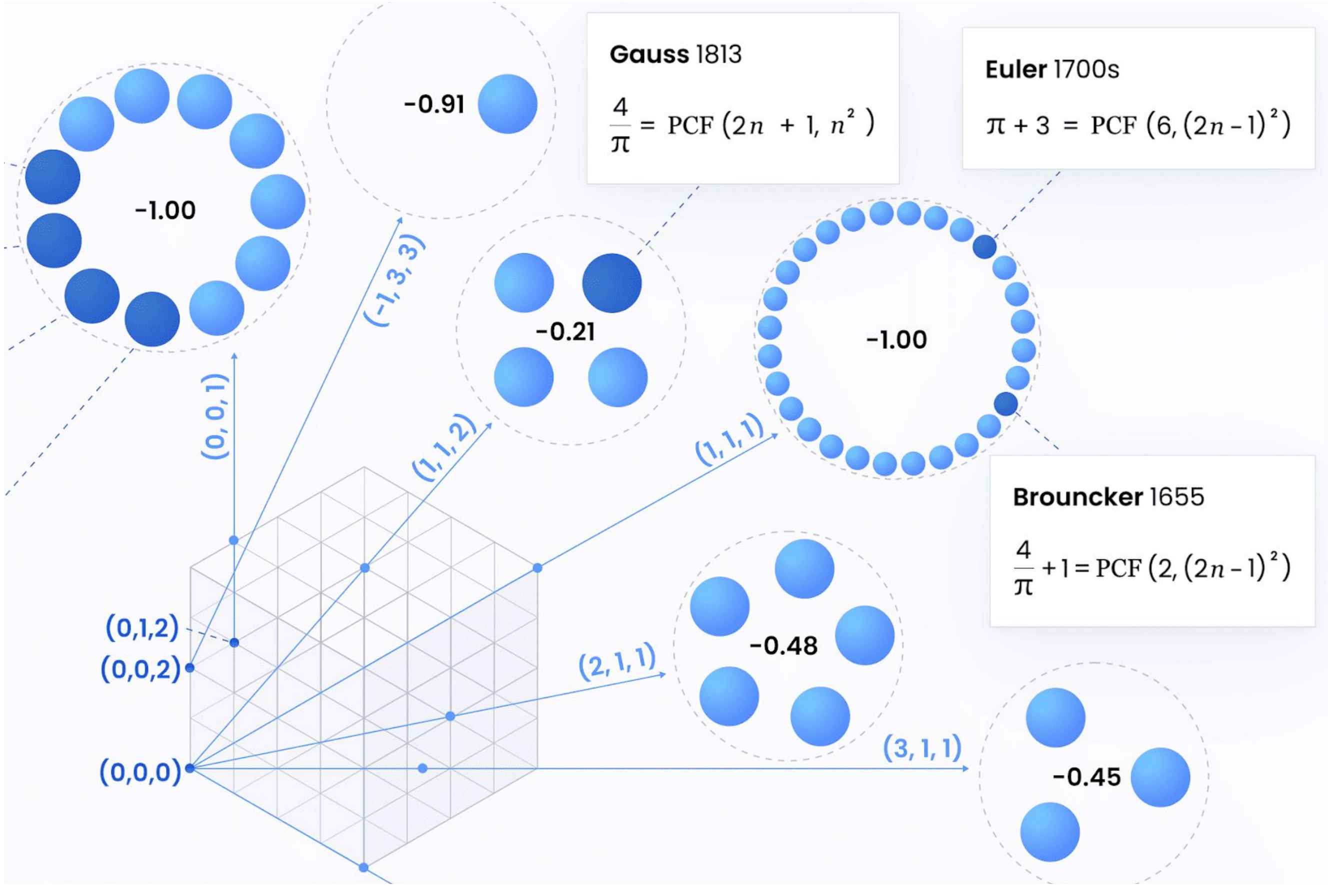
$$ {}_2F_1(a,b;c;z) = \sum_{n=0}^{\infty} \frac{(a)_n\,(b)_n}{(c)_n}\frac{z^n}{n!} $$

$$ (a)_n = \frac{\Gamma(a+n)}{\Gamma(a)} $$



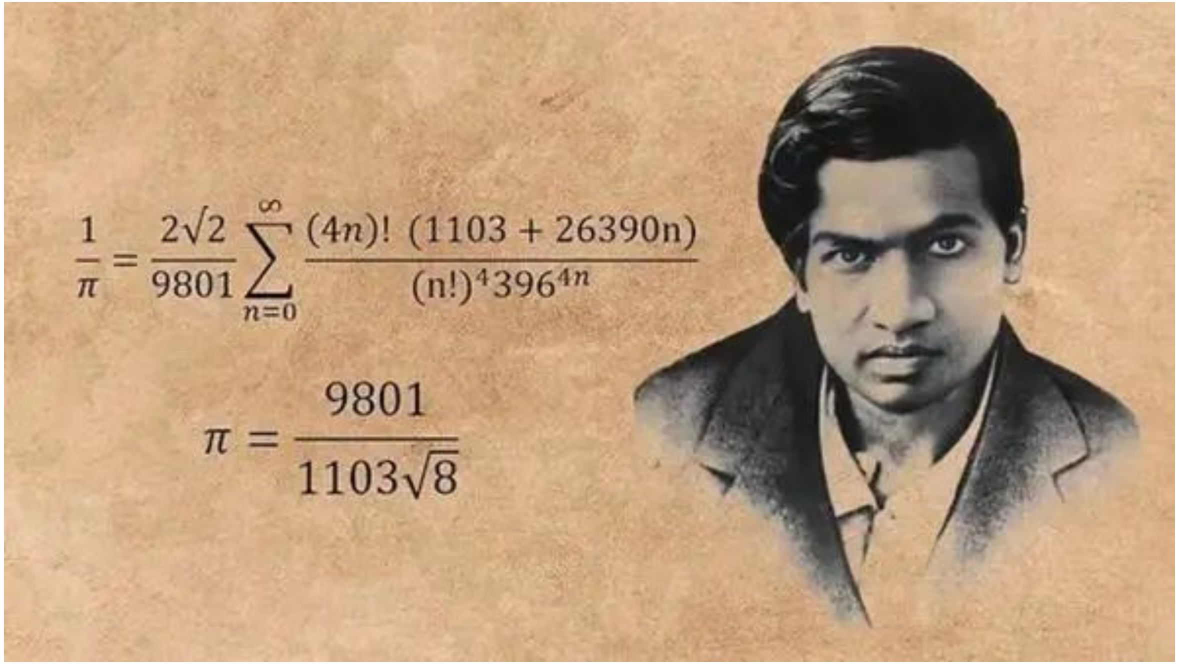
Elimelech et al. “*Algorithm-assisted discovery of an intrinsic order among mathematical constants*” (PNAS 2024)
Weinbaum et al. “*On Conservative Matrix Fields: Continuous Asymptotics and Arithmetic*” (arXiv:2507.08138)

43% (166) of Known Formulas Unified



Tomer Raz et al. *"From Euler to AI: Unifying formulas for mathematical constants"* (NeurIPS 2025)

Ramanujan (1914)



Modular equations and approximations to π

Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

Ramanujan (1914)

Mathematicians find one pi formula to rule them all

Mathematicians find one pi formula to rule them all

A mixture of AI and algorithms uncovered a hidden structure spanning 2,000 years of equations for pi

BY LYNDIE CHIOU EDITED BY CLARA MOSKOWITZ

Tomer Raz et al. "*From Euler to AI: Unifying formulas for mathematical constants*" (NeurIPS 2025)

Ramanujan Formulas - Unified

#	formula	shift (a_2, a_3, a_4)	z	A	B
1	$\frac{4}{\pi} = \sum_{n\geq 0}(6n+1)\binom{2n}{n}^3 \frac{1}{2^{6n}}$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$\frac{1}{4}$	1	6
2	$\frac{16}{\pi} = \sum_{n\geq 0}(42n+5)\binom{2n}{n}^3 \frac{1}{2^{12n}}$	$(\frac{1}{2}, \frac{1}{2}, \frac{1}{2})$	$\frac{1}{2^6}$	5	42
	$\cdots$				
16	$\frac{2}{\pi\sqrt{11}} = \sum_{n\geq 0}(280n+19)\binom{2n}{n}^2\binom{4n}{2n}\frac{1}{2^{8n}99^{2n+1}}$	$(\frac{1}{4}, \frac{1}{2}, \frac{3}{4})$	$\frac{1}{99^2}$	19	280
17	$\frac{1}{2\pi\sqrt{2}} = \sum_{n\geq 0}(26390n+1103)\binom{2n}{n}^2\binom{4n}{2n}\frac{1}{2^{8n}99^{4n+2}}$	$(\frac{1}{4}, \frac{1}{2}, \frac{3}{4})$	$\frac{1}{99^4}$	1103	26390

Human Hunch: Sum Elements Look Like 3F2

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!} $$

Human Hunch: Sum Elements Look Like 3F2

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!} $$

“Just ask an LLM”

$$ \frac{4}{\pi} = \sum_{n \geq 0} (6n+1) \binom{2n}{n}^3 \frac{1}{2^{6n}} $$

$$ \Downarrow $$

$$ \frac{4}{\pi} = {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, 1; 1\right) + \frac{3}{4} {}_3F_2\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 2, 2; 1\right) $$

Human Hunch: Sum Elements Look Like 3F2

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!} $$

"Just ask an LLM"

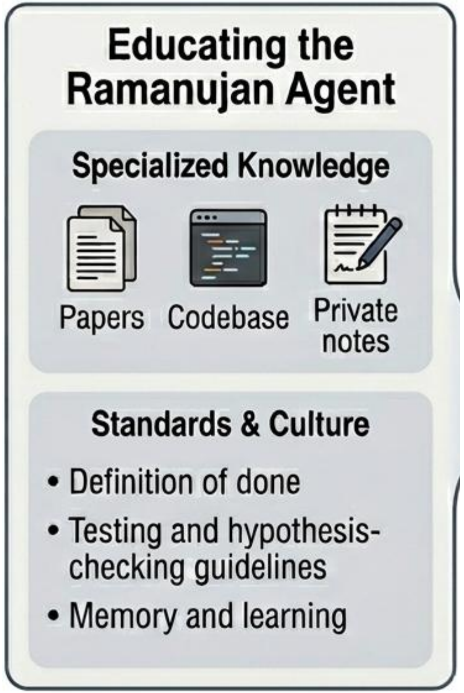
$$ \frac{4}{\pi} = \sum_{n\geq 0} (6n+1) \binom{2n}{n}^3 \frac{1}{2^{6n}} $$

$$ \Downarrow $$

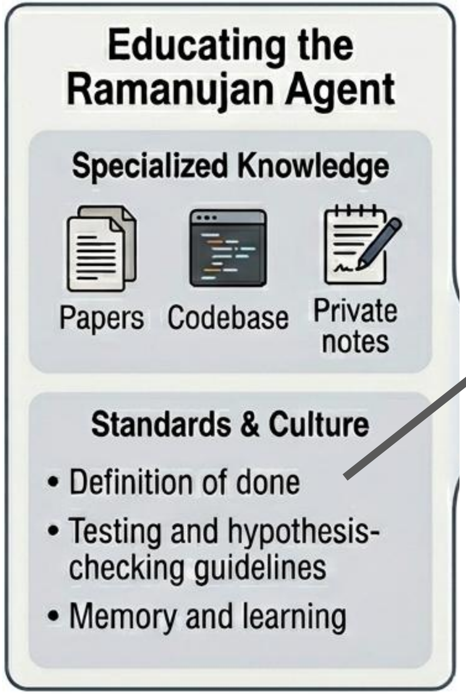
$$ \frac{4}{\pi} = {}_3F_2\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}; 1, 1; 1\right) + \frac{3}{4} {}_3F_2\left(\frac{3}{2}, \frac{3}{2}, \frac{3}{2}; 2, 2; 1\right) $$

Correct, but useless…

Ramanujan Agent

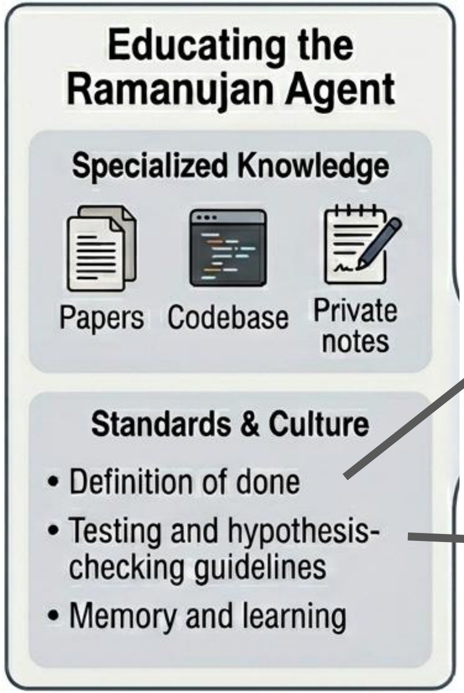


Ramanujan Agent



- Numerical verification to 100+ digits at depth ≥ 1000.
- Convergence rate measured empirically.
- …

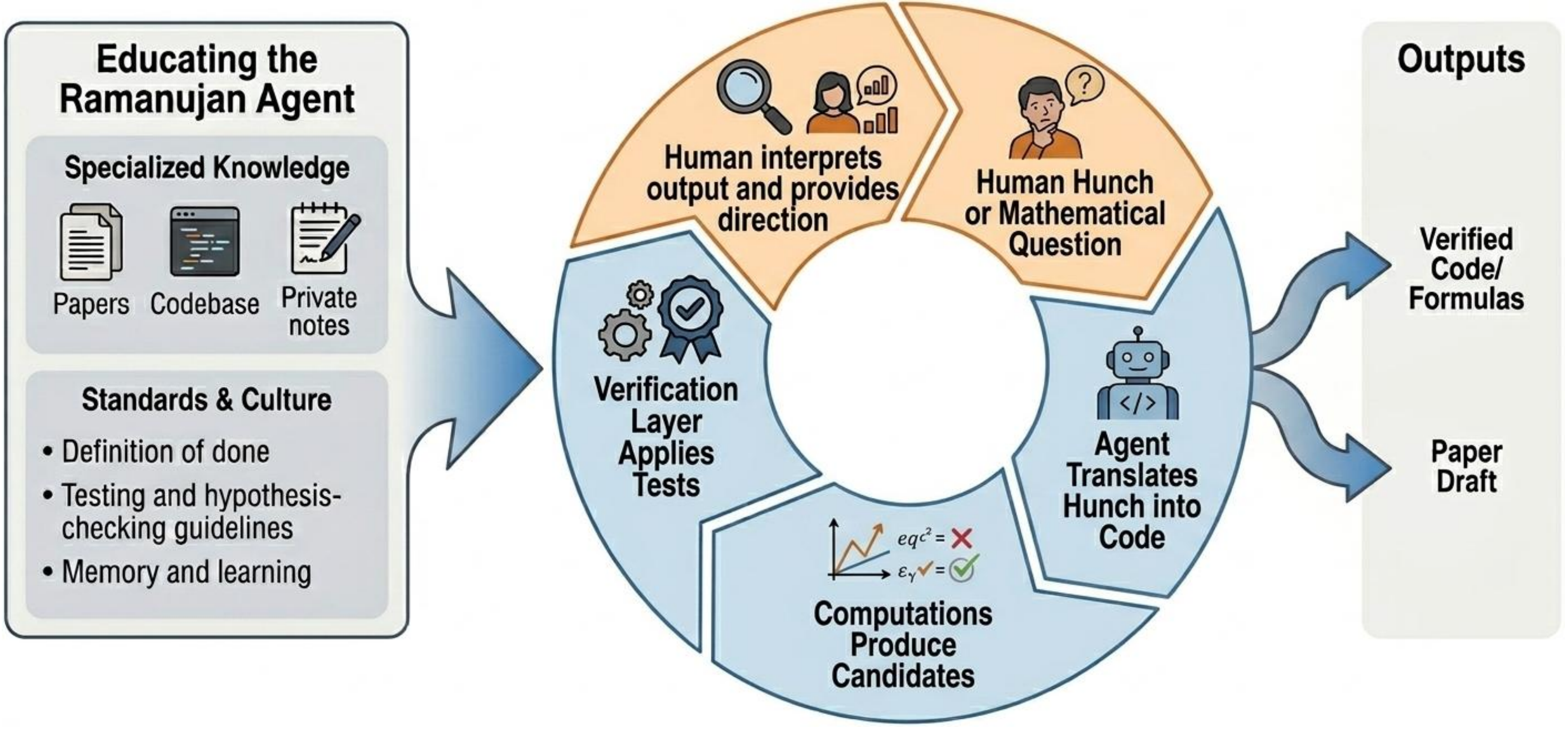
Ramanujan Agent



- Numerical verification to 100+ digits at depth ≥ 1000.
- Convergence rate measured empirically.
- …

- Test hypothesis on multiple, meaningfully different, cases.
- Flag uncertainties. If something is conjectured but not proven, say so explicitly.
- …

Hybrid Research Methodology



Agent: Use ramanujantools To Scan 3F2 CMFs

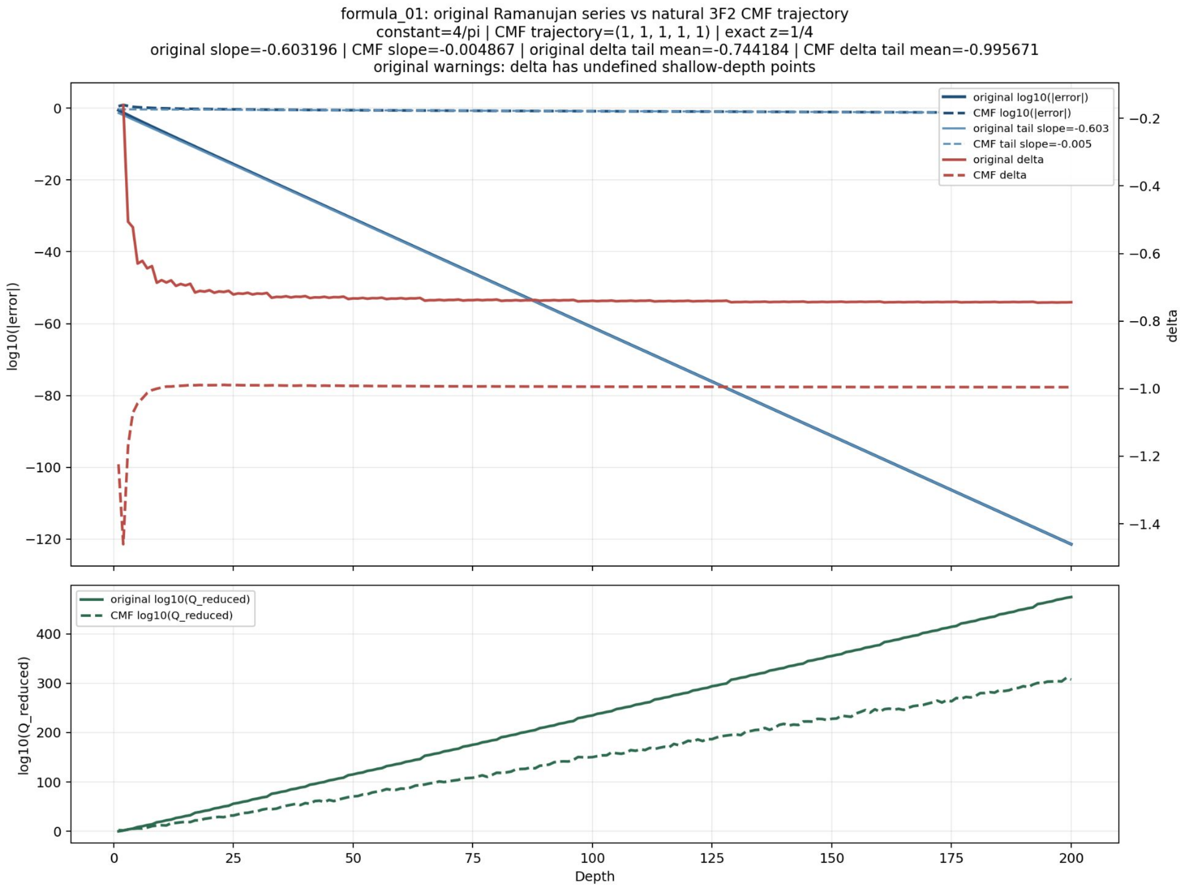
Ramanujan Machine Group, *“ramanujantools”*, https://github.com/RamanujanMachine/ramanujantools (2025)

Agent: Use ramanujantools To Scan 3F2 CMFs

1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 1], ["y	13	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 6], ["y	13	200	320	4/pi	1.27324	3.31E-14	-0.98893	-1.0005	-1.0004	0.256312	0.00231	0.002981	5.873028	0.991869	TRUE	FALSE	ok	["log-erro	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 3], ["x2", 6], ["y	13	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 3], ["x2", 6], ["y	13	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 2], ["x2", 4], ["y	14	200	320												FALSE	FALSE	failed	[]	[]	identify_t	Limit.iden	Li
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 4], ["x2", 4], ["y	14	200	320	1	1	0.27324	#NAME?								FALSE	FALSE	ok	["log_errc	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 4], ["y	14	200	320	4/pi	1.27324	2.78E-14	-0.98438	-0.99958	-1.00128	0.255399	0.005998	0.004124	4.340764	0.984769	TRUE	FALSE	ok	["log-erro	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 4], ["x2", 4], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 4],	[["x0", 4], ["x1", 4], ["x2", 4], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 5], ["y	14	200	320	4/pi	1.27324	1.04E-14	-0.98712	-1.00048	-0.99681	0.256291	-0.01701	0.045925	5.01576	0.983528	TRUE	FALSE	ok	["log-erro	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 2], ["x2", 5], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 3], ["x2", 5], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 2], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 5], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 5], ["x2", 5], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 3], ["y	14	200	320												FALSE	FALSE	failed	[]	[]	identify_t	Limit.iden	Li
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 6], ["y	14	200	320	4/pi	1.27324	4.66E-15	-0.98798	-0.99954	-0.99965	0.255351	-0.00287	0.002083	5.345825	0.986947	TRUE	FALSE	ok	["log-erro	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 2], ["x2", 6], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 4], ["x2", 6], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 6], ["y	14	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 3],	[["x0", 3], ["x1", 3], ["x2", 3], ["y	15	200	320	4/pi	1.27324	2.09E-14	-0.98558	-0.99846	-0.99894	0.25428	-0.00458	0.999869	4.45963	0.981776	TRUE	FALSE	ok	["log_errc	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 4], ["x2", 4], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 3], ["x2", 4], ["y	15	200	320	4/pi	1.27324	4.60E-13	-0.98671								TRUE	FALSE	tail_diagn	["log_errc	["The log(	tail_diagn	ValueErro	Va
1	4/pi	-0.74418	0.05	[["x0", 3],	[["x0", 3], ["x1", 3], ["x2", 4], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 1], ["x2", 3], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 2], ["x2", 3], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 3], ["x2", 4], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 1],	[["x0", 1], ["x1", 4], ["x2", 4], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 2], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 2], ["x2", 5], ["y	15	200	320	4/pi	1.27324	1.55E-14	-0.98736	-0.99978	-1.00089	0.255597	0.006398	0.008633	5.221586	0.99133	TRUE	FALSE	ok	["log-erro	["The log(error_n) fit against n is			
1	4/pi	-0.74418	0.05	[["x0", 2],	[["x0", 2], ["x1", 3], ["x2", 5], ["y	15	200	320												FALSE	FALSE	screen_fa	[]	[]	screen_identify_pi		

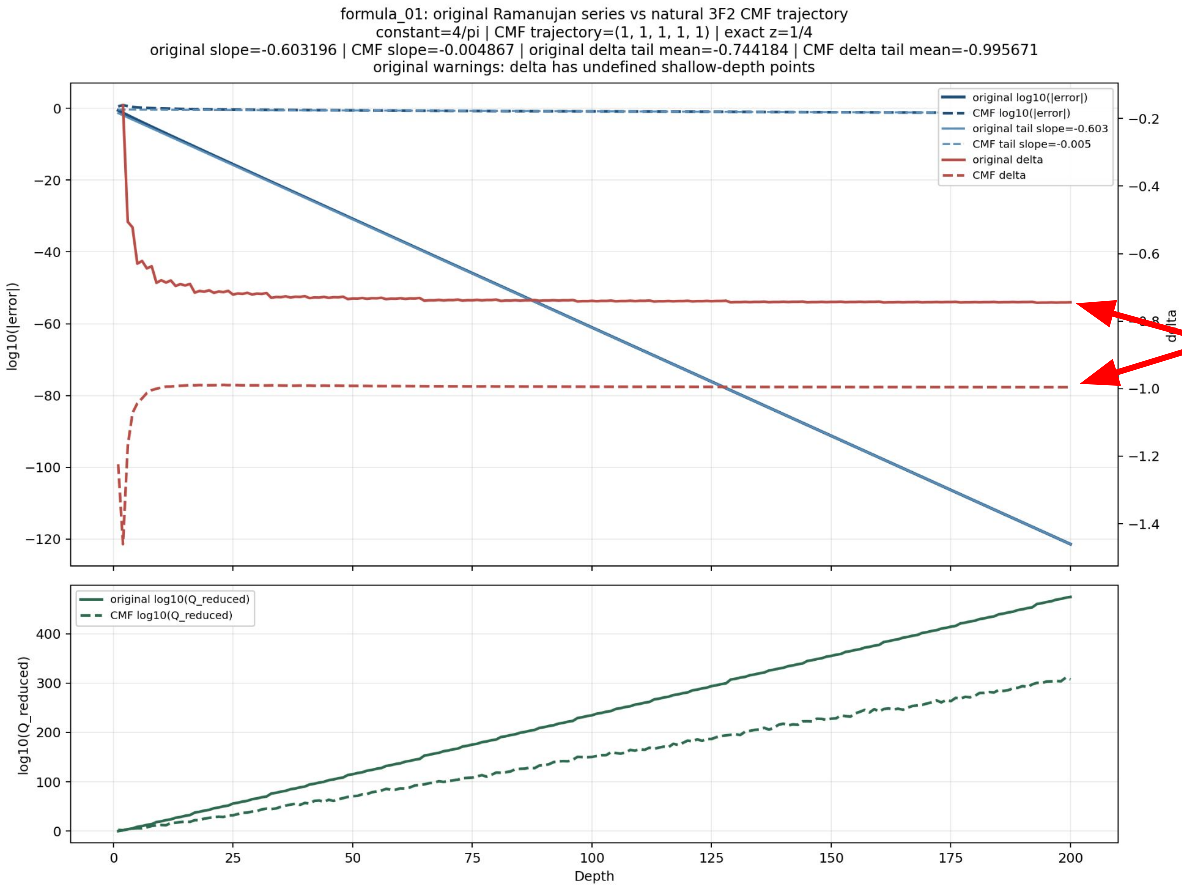
Ramanujan Machine Group, *"ramanujantools"*, https://github.com/RamanujanMachine/ramanujantools (2025)

Agent: Analysis of Best Candidates



Shalyt et al. *"Unsupervised discovery of formulas for mathematical constants"* (NeurIPS 2024)

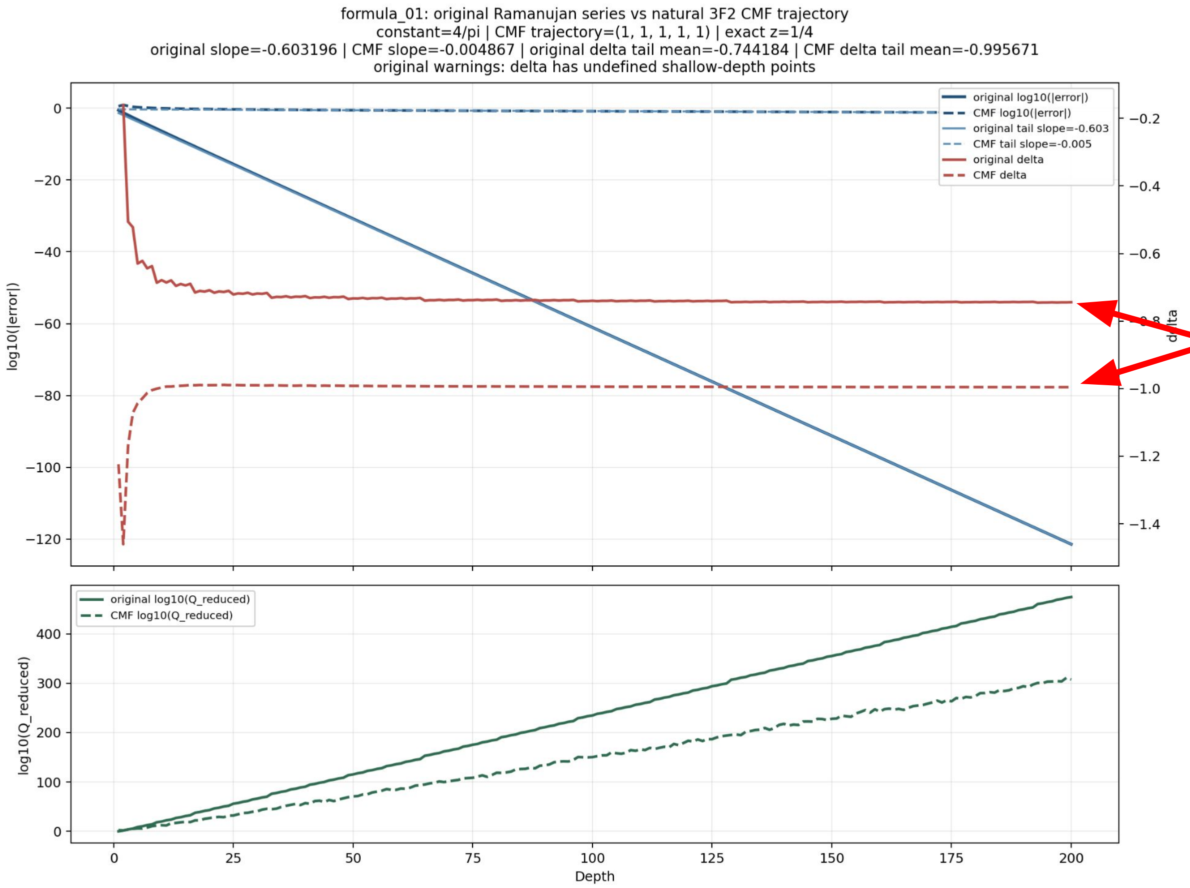
Human: Visual Debugging With Dynamic Metrics



Same limit but different irrationality measure => NOT the same formula.

Shalyt et al. *"Unsupervised discovery of formulas for mathematical constants"* (NeurIPS 2024)

Human: Visual Debugging With Dynamic Metrics



Same limit but different irrationality measure => NOT the same formula.

Add as a guardrails!

Shalyt et al. *"Unsupervised discovery of formulas for mathematical constants"* (NeurIPS 2024)

Human: 3F2 => 4F3

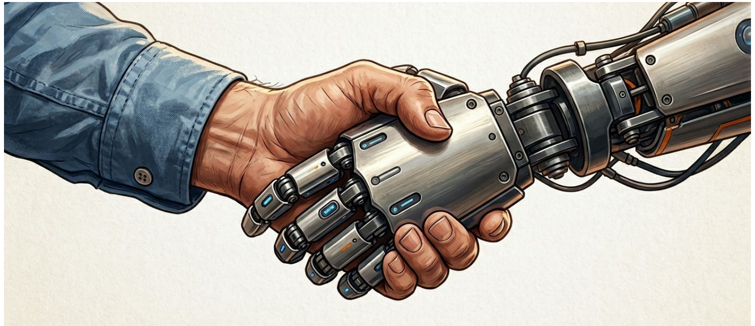
The tail of the Ramanujan sums behaves like 4F3 - the 3F2 was a red herring.

Human: 3F2 => 4F3

The tail of the Ramanujan sums behaves like 4F3 - the 3F2 was a red herring.

Agent: Generalization From 1 Formula To All 17

Hybrid Research Methodology



Human	Agent
3F2 Hunch.	Broad CMF Scans.
Defining task-specific guardrails and auto-validations.	Summarizing experimental results and choosing best candidates.
3F2 -> 4F3 realization.	Generalization from a single successfully identified formula.

Result: All 17 Formulas Are In 4F3-Based CMFs

Each formula trajectory is along the vector (0, 1, 1, 1; 1, 1, 1). The initial point is (1, a2, a3, a4; 1, 1, 1). A, B are seed values.

#	formula	shift (a_2, a_3, a_4)	z	A	B
1	$\frac{4}{\pi} = \sum_{n\geq 0}(6n+1)\binom{2n}{n}^3 \frac{1}{2^{6n}}$	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{4}$	1	6
2	$\frac{16}{\pi} = \sum_{n\geq 0}(42n+5)\binom{2n}{n}^3 \frac{1}{2^{12n}}$	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{2^6}$	5	42
	$\dots$				
16	$\frac{2}{\pi\sqrt{11}} = \sum_{n\geq 0}(280n+19)\binom{2n}{n}^2\binom{4n}{2n} \frac{1}{2^{8n}99^{2n+1}}$	$\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$	$\frac{1}{99^2}$	19	280
17	$\frac{1}{2\pi\sqrt{2}} = \sum_{n\geq 0}(26390n+1103)\binom{2n}{n}^2\binom{4n}{2n} \frac{1}{2^{8n}99^{4n+2}}$	$\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$	$\frac{1}{99^4}$	1103	26390

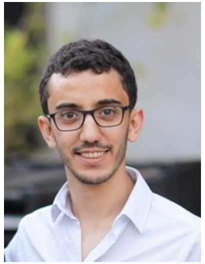
Technion
Israel Institute of Technology

Ramanujan Machine


Prof. Ido Kaminer


Michael Shalyt


Elyasheev Leibtag


Rotem Elimelech


Rotem Kalisch


Shachar Weinbaum


Tali Monderer

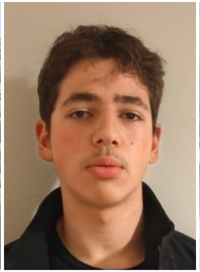

Hila Barkan

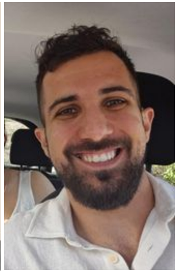

Uri Hitin


Ashvni Narayanan


Tomer Raz


Ilay Menahem


Zeev Ben Yehuda


Stav Bley

ramanujanmachine.com

github.com/RamanujanMachine/RamanujanAgent

Ramanujan (1914)

Modular equations and approximations to π

Quarterly Journal of Mathematics, XLV, 1914, 350 – 372

$$\frac{4}{\pi} = \sum_{n\geq 0} (6n+1)\binom{2n}{n}^3 \frac{1}{2^{6n}}$$

$$\frac{1}{\pi} = \frac{2\sqrt{2}}{9801}\sum_{k=0}^{\infty}\frac{(4k)!(1103+26390k)}{(k!)^4 396^{4k}}$$

Ramanujan (1914)

$$\frac{4}{\pi} = \frac{3}{2} - \frac{23}{2^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{43}{2^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} - \cdots, \tag{35}$$

$$\frac{4}{\pi\sqrt{3}} = \frac{3}{4} - \frac{31}{3\cdot 4^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{59}{3^2\cdot 4^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} - \cdots, \tag{36}$$

$$\frac{4}{\pi} = \frac{23}{18} - \frac{283}{18^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{543}{18^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} - \cdots, \tag{37}$$

$$\frac{4}{\pi\sqrt{5}} = \frac{41}{72} - \frac{685}{5\cdot 72^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{1329}{5^2\cdot 72^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} - \cdots, \tag{38}$$

$$\frac{4}{\pi} = \frac{1123}{882} - \frac{22583}{882^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{44043}{882^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} - \cdots, \tag{39}$$

$$\frac{2\sqrt{3}}{\pi} = 1 + \frac{9}{9}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{17}{9^2}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} + \cdots, \tag{40}$$

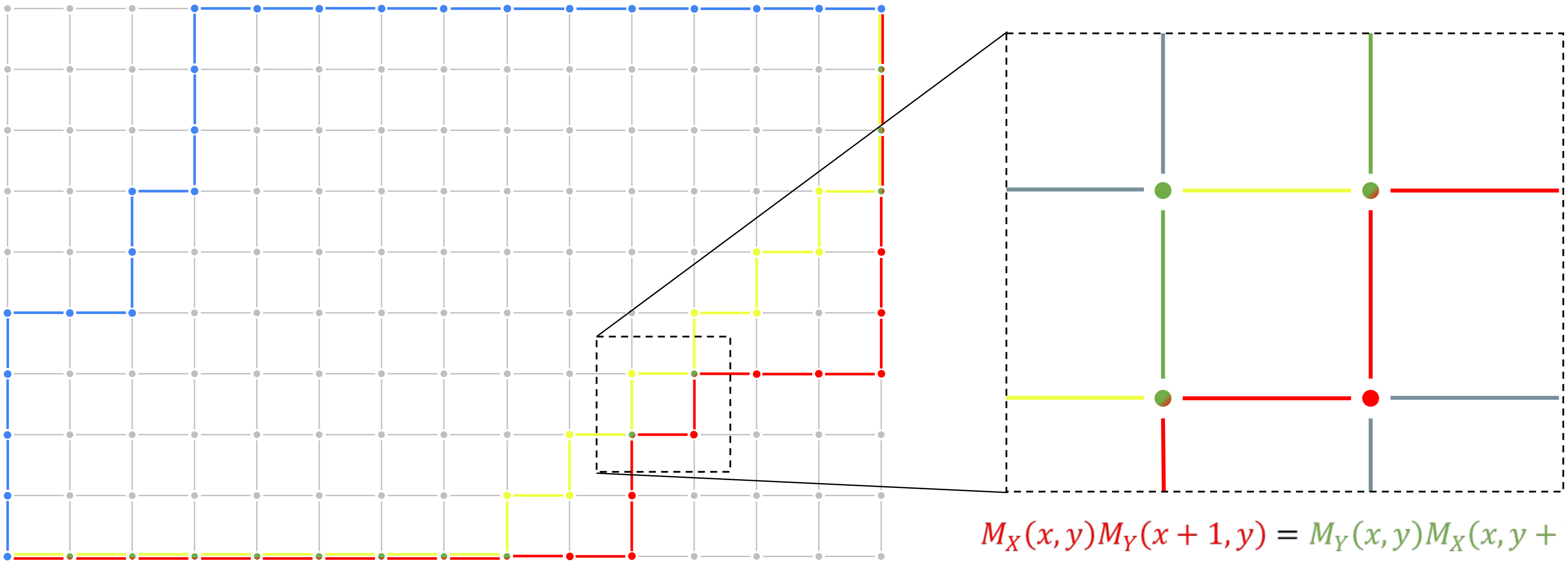
$$\frac{1}{2\pi\sqrt{2}} = \frac{1}{9} + \frac{11}{9^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{21}{9^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} + \cdots, \tag{41}$$

$$\frac{1}{3\pi\sqrt{3}} = \frac{3}{49} + \frac{43}{49^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{83}{49^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} + \cdots, \tag{42}$$

$$\frac{2}{\pi\sqrt{11}} = \frac{19}{99} + \frac{299}{99^3}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{579}{99^5}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} + \cdots, \tag{43}$$

$$\frac{1}{2\pi\sqrt{2}} = \frac{1103}{99^2} + \frac{27493}{99^6}\frac{1}{2}\frac{1\cdot 3}{4^2} + \frac{53883}{99^{10}}\frac{1\cdot 3}{2\cdot 4}\frac{1\cdot 3\cdot 5\cdot 7}{4^2\cdot 8^2} + \cdots. \tag{44}$$

Conservative Matrix Fields



$$M_X(x,y)M_Y(x+1,y) = M_Y(x,y)M_X(x,y+1)$$

Elimelech et al. "*Algorithm-assisted discovery of an intrinsic order among mathematical constants*" (PNAS 2024)

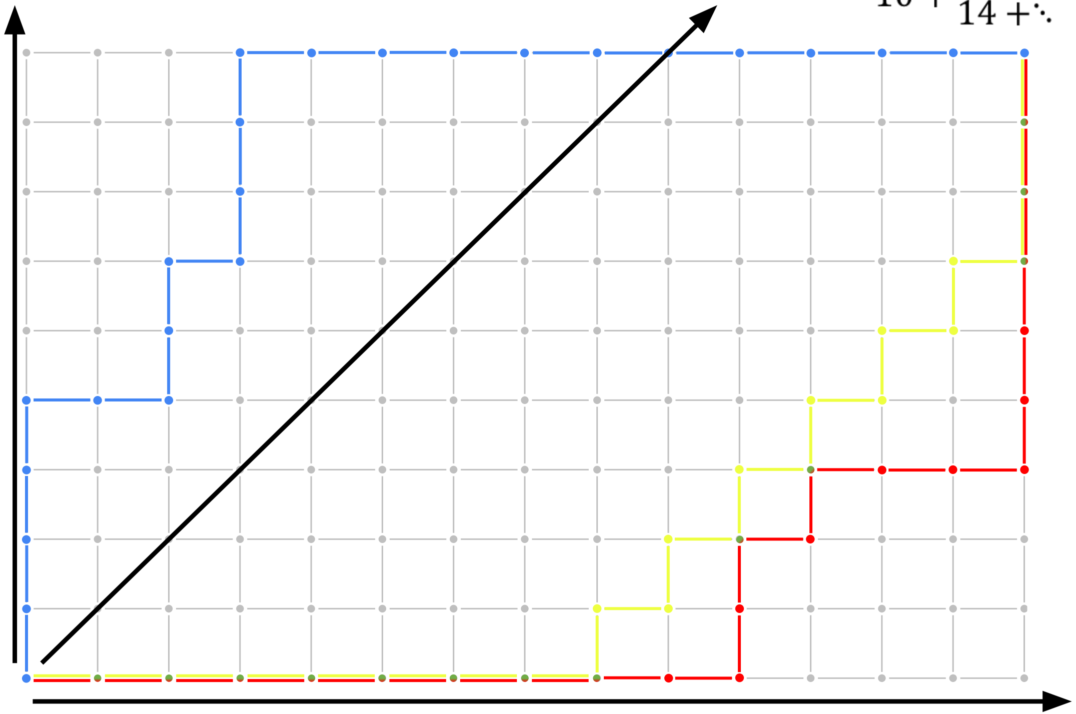
Conservative Matrix Fields

Fast convergence

$$\frac{1}{e-1} = 1/2 + \cfrac{1/2}{6 + \cfrac{1}{10 + \cfrac{1}{14 + \ddots}}}$$

Slow convergence

$$\frac{1}{e-1} = \cfrac{1}{1 + \cfrac{2}{2 + \cfrac{3}{3 + \ddots}}}$$



$$M_x(x,y) = \begin{bmatrix} 1 & -y-1 \\ -1 & x+y+2 \end{bmatrix}$$

$$M_y(x,y) = \begin{bmatrix} 0 & -y-1 \\ -1 & x+y+1 \end{bmatrix}$$

Slow convergence

$$\frac{1}{e-1} = 1 - \cfrac{1}{3 - \cfrac{2}{4 - \cfrac{3}{5 - \ddots}}}$$

Elimelech et al. "*Algorithm-assisted discovery of an intrinsic order among mathematical constants*" (PNAS 2024)

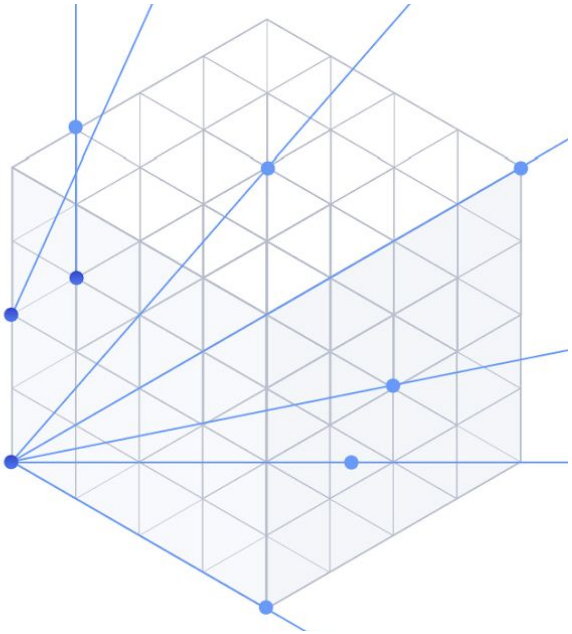
Formula Unification

$$M_x = \begin{pmatrix} 1 & y \\ \frac{1}{x} & 1 + \frac{x+y-2z+2}{x} \end{pmatrix}$$

$$M_y = \begin{pmatrix} 1 & x \\ \frac{1}{y} & 1 + \frac{x+y-2z+2}{y} \end{pmatrix}$$

$$M_z = \begin{pmatrix} -\frac{z(x+y-z)}{(x-z)(y-z)} & \frac{xyz}{(x-z)(y-z)} \\ \frac{z}{(x-z)(y-z)} & -\frac{z^2}{(x-z)(y-z)} \end{pmatrix}$$

$$4,\ 3,\ \frac{19}{6},\ \frac{160}{51},\ \frac{1744}{555}\cdots$$



$$4,\ \frac{8}{3},\ \frac{52}{15},\ \frac{304}{105},\ \frac{1052}{315}\cdots$$

Tomer Raz et al. "*From Euler to AI: Unifying formulas for mathematical constants*" (NeurIPS 2025)

Ramanujan Formulas Unification

$$\frac{K}{\pi} = \sum_{n=0}^{\infty} \sigma\,(A + B\,n)\ \frac{(a_0)_n\ (a_1)_n\ (a_2)_n}{(n!)^3}\ z_0^{\,n}$$

$$(a)_n = a(a+1)(a+2)\cdots(a+n-1)$$

Human: Looks Hypergeometric

$$ {}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!} $$

Human: Looks Hypergeometric

$$
{}_3F_2(a_1, a_2, a_3; b_1, b_2; z) = \sum_{n=0}^{\infty} \frac{(a_1)_n (a_2)_n (a_3)_n}{(b_1)_n (b_2)_n} \frac{z^n}{n!}
$$

$$
\frac{K}{\pi} = \sum_{n=0}^{\infty} \sigma \left(A + B\, n\right) \underbrace{\frac{(a_0)_n \; (a_1)_n \; (a_2)_n}{(n!)^3} \; z_0^n}_{{}_3F_2\left(\left.\begin{matrix} a_0, \; a_1, \; a_2 \\ 1, \; 1 \end{matrix} \right| z_0\right)}
$$

AI: Translate All Summands Into 3F2

$$\frac{4}{\pi} = \sum_{n\geq 0}(6n+1)\binom{2n}{n}^3 \frac{1}{2^{6n}}$$

$$\downarrow$$

$$\frac{4}{\pi} = \sum_{n=0}^{\infty}(6n+1)\frac{\left(\frac{1}{2}\right)_n^3}{(n!)^3}$$

$$\downarrow$$

$$\frac{4}{\pi} = {}_3F_2\left(\frac{1}{2},\frac{1}{2},\frac{1}{2};1,1;1\right) + \frac{3}{4}{}_3F_2\left(\frac{3}{2},\frac{3}{2},\frac{3}{2};2,2;1\right)$$

Human: 3F2 => 4F3

Tail sum:

$$R_n = \sum_{k=n}^{\infty} \sigma(A + Bk) \frac{(a_0)_k (a_1)_k (a_2)_k}{(k!)^3} z_0^k$$

$$k = n + m$$

Human: 3F2 => 4F3

$$R_n = \sigma \frac{(a_0)_n (a_1)_n (a_2)_n}{(n!)^3} z_0^n \left(A + B(n+\theta)\right) \cdot$$

$$\cdot \sum_{m=0}^{\infty} \frac{(a_0+n)_m (a_1+n)_m (a_2+n)_m}{(n+1)_m^3} z_0^m$$

Human: 3F2 => 4F3

$$R_n = \sigma \frac{(a_0)_n (a_1)_n (a_2)_n}{(n!)^3} z_0^n \left(A + B(n+\theta)\right)\cdot$$

$$\cdot \underbrace{\sum_{m=0}^{\infty} \frac{(a_0+n)_m (a_1+n)_m (a_2+n)_m}{(n+1)_m^3} z_0^m}_{{}_4F_3\left(\begin{matrix}1,\ a_0+n,\ a_1+n,\ a_2+n \\ n+1,\ n+1,\ n+1\end{matrix}; z_0\right)}$$

AI: Apply The Method To All 17 Formulas

Each formula trajectory is along the vector (0, 1, 1, 1; 1, 1, 1).
The initial point is (1, a2, a3, a4; 1, 1, 1). A, B are seed values.

AI: Apply The Method To All 17 Formulas

Each formula trajectory is along the vector (0, 1, 1, 1; 1, 1, 1).
The initial point is (1, a2, a3, a4; 1, 1, 1). A, B are seed values.

#	formula	shift (a_2, a_3, a_4)	z	A	B
1	$\frac{4}{\pi} = \sum_{n\geq 0}(6n+1)\binom{2n}{n}^3 \frac{1}{2^{6n}}$	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{4}$	1	6
2	$\frac{16}{\pi} = \sum_{n\geq 0}(42n+5)\binom{2n}{n}^3 \frac{1}{2^{12n}}$	$\left(\frac{1}{2}, \frac{1}{2}, \frac{1}{2}\right)$	$\frac{1}{2^6}$	5	42
	$\cdots$				
16	$\frac{2}{\pi\sqrt{11}} = \sum_{n\geq 0}(280n+19)\binom{2n}{n}^2\binom{4n}{2n}\frac{1}{2^{8n}99^{2n+1}}$	$\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$	$\frac{1}{99^2}$	19	280
17	$\frac{1}{2\pi\sqrt{2}} = \sum_{n\geq 0}(26390n+1103)\binom{2n}{n}^2\binom{4n}{2n}\frac{1}{2^{8n}99^{4n+2}}$	$\left(\frac{1}{4}, \frac{1}{2}, \frac{3}{4}\right)$	$\frac{1}{99^4}$	1103	26390

Dynamical Metrics

$$\epsilon(n) := \left|\frac{p_n}{q_n} - L\right| \longrightarrow \epsilon(n) \sim n!^{\eta} \cdot e^{\gamma n} \cdot n^{\beta}$$

$$\left|L - \frac{p_n}{q_n}\right| < \frac{1}{q_n^{1+\delta}}$$

$$\delta_n = \frac{-\log\left|L - \frac{p_n}{q_n}\right|}{\log|\tilde{q}_n|} - 1 \quad , \quad \tilde{q}_n = \frac{q_n}{\gcd(p_n, q_n)}$$

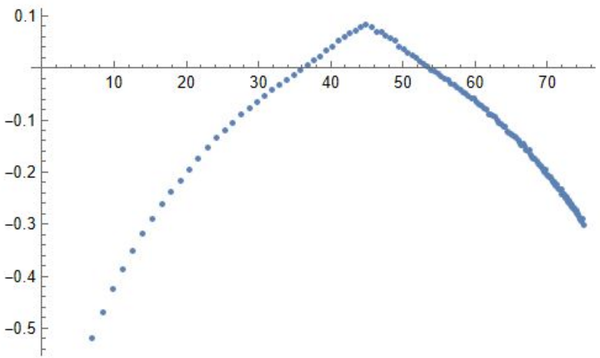
$$\tilde{q}_n \sim n!^{\eta'} \cdot e^{\gamma' n} \cdot n^{\beta'}$$

Shalyt et al. "*Unsupervised Discovery of Formulas for Mathematical Constants*" (NeurIPS 2024)

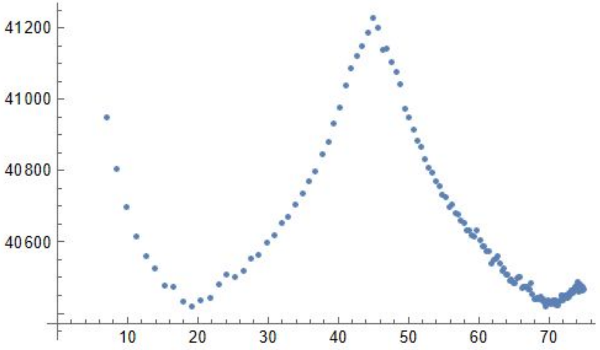
Scanning CMFs - Continuity of Metrics Vs. Angle

- ζ(3) CMF.
  - Reproducing Apery's irrationality-proving sequence.
- Metrics seem (empirically) to be continuous as function of angle.
  - Convergence rate.
  - Irrationality measure.
  - Sequence eigenvalues.
  - Etc.
- Next big goal: ζ(5) irrationality.

Irrationality Measure Vs. Angle



Log[GCD] Vs. Angle



Scanning CMFs - Continuity of Metrics Vs. Angle

- π CMF.
  - Best convergence rate is at 45°.
  - Best irrationality measure is at ~63°, presumably arctan[2].